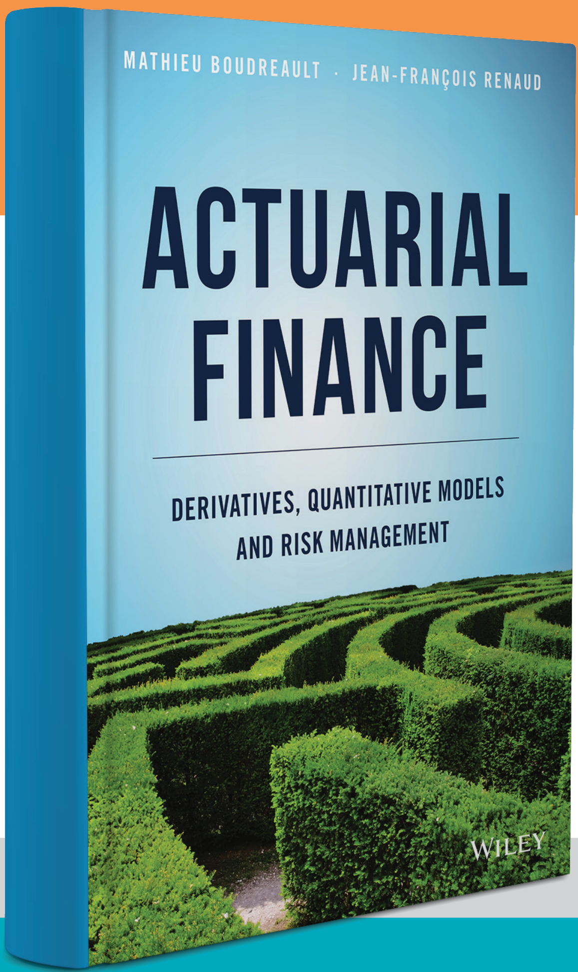

فراخوان ترجمه کتاب

پژوهشکده بیمه

پژوهشکده بیمه، به منظور کمک به گسترش دانش بیمه‌ای، ترجمه کتاب

Actuarial Finance:
Derivatives, Quantitative Models and Risk Managment

را در دستور کار خود قرار داده است. لذا از کلیه اساتید، پژوهشگران، صاحبنظران و کارشناسان دعوت می‌شود که در صورت تمایل به ترجمه کتاب مذکور، کاربرگ درخواست ترجمه پیوست را به همراه سوابق علمی و اجرایی خود و ترجمه صفحات ذکر شده با ذکر عنوان کتاب، حداکثر تا تاریخ ۱۴۰۳/۰۱/۳۰ به آدرس ایمیل nashr@irc.ac.ir ارسال فرمایند.

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3

Forwards and futures

In many situations, it is good risk management to fix today the price at which an asset will be purchased in the future. Here are two examples:

- Consider a company regularly buying gold as an input for its business, e.g. that company could transform gold to make jewelry. This company is clearly exposed to the (random) fluctuations of the gold price. More specifically, it would suffer from an increase in the price. In this case, it may be a good idea to enter into a financial agreement that fixes the price of gold today for delivery in the future. Therefore, the cost of jewelry made over the next few weeks/months can be known in advance.
- An insurance company based in the U.S. but also doing business in Canada is exposed to changes in the currency rate between the U.S. dollar (USD) and the Canadian dollar (CAD). It could enter into a financial agreement to fix today the exchange rate between these currencies that will apply in the future.

Forward contracts and futures contracts can be used specifically for this purpose: fix today the price of a good to be bought in the future. They are also commonly known as *forwards* and *futures*, without the word contract or agreement attached to them.

The role of this chapter is to provide an introduction to forwards and futures. The specific objectives are to:

- recognize situations where forward contracts and futures contracts can be used to manage risks;
- understand the difference between a forward contract and a futures contract;
- replicate the cash flows of forward contracts written on stocks or on foreign currencies;
- calculate the forward price of stocks and of foreign currencies;
- calculate the margin balance on long and short positions of futures contracts.

3.1 Framework

This section lays the foundations of forwards and futures for the rest of the chapter. The content applies regardless of the underlying asset.

3.1.1 Terminology

A **forward contract** or a **futures contract** is a contract that engages one party to buy (and the other to sell) an asset some time in the future for a price determined today at inception of the

Actuarial Finance: Derivatives, Quantitative Models and Risk Management, First Edition.
Mathieu Boudreault and Jean-François Renaud.

contract. That time in the future is known as the expiration, **maturity** or **delivery date** whereas the said price is known as the **delivery price**.

The most important difference between forwards and futures is that futures are standardized and exchange-traded. Exchanges require investors to hold money aside to protect both sides of the transaction from default risk. This is usually known as the process of *marking-to-market*. This difference aside, forwards and futures serve the same purpose: fixing today the price at which an asset will be bought in the future.

Forward contracts and futures contracts also differ from *spot contracts*:

- A spot contract is agreed upon today between a buyer and a seller, the asset is paid for and delivered (almost) immediately.
- A forward (futures) contract is also agreed upon today but the asset will be paid for and delivered at maturity of the forward (futures) contract.

Therefore, if you need to acquire the asset, say in 3 months, then instead of waiting 3 months and buying the asset on the spot market and paying a price viewed as random today, the forward contract fixes that price today.

To simplify the presentation of this chapter, we will start by looking at forward contracts and we will come back to futures contracts in Section 3.5.

3.1.2 Notation

Let us begin with the following standard notation:

- K is the delivery price;
- T is the maturity date.

We say that delivery of the underlying asset will occur at time T or that the forward contract will *mature at time* T.

Entering the long position of a forward contract, or said differently *buying a forward contract*, does not usually require an initial payment/premium. However, to avoid arbitrage opportunities, this restriction will have an impact on the *right value* of K. For now, we will consider that K can take any value and that entering a forward contract might require, or not, an upfront payment. We will come back to this issue later when we discuss the *forward price*.

The cash flows of a forward contract are:

- At inception (at time 0), both parties agree on K (and on T) and an up-front premium might be required (paid by the forward buyer (long position) to the forward seller (short position)), or vice versa.
- At maturity (at time T), the investor with the long position pays K and receives the underlying asset worth S_T from the investor with the short position, no matter what the realization of the random variable S_T is. This is illustrated in Figure 3.1.

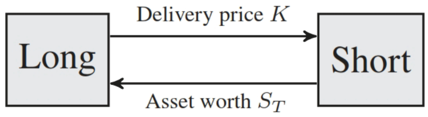

Figure 3.1 Cash flows of a forward contract at maturity

Example 6.4.4 *Put-call parity on a stock index*
The S&P 500 index is currently at 1000. A 6-month option to sell the index for 990 currently trades for 25 whereas a similar option to buy the index trades for 50. You also know that the (average) dividend yield generated by the stocks in the index is 3% (continuously compounded). What is the price of a 6-month Treasury zero-coupon bond?

Let B_0 be the initial price of a 6-month zero-coupon bond (with face value 1). Using the put-call parity of equation (6.4.6), we get

$$50 - 25 = 1000e^{-0.03\times 0.5} - 990 \times B_0.$$

We get $B_0 = 0.9698$. ■

6.5 American options

As discussed in Chapter 5, European and American options differ on *when* the investor is allowed to exercise its option, i.e. when she can buy or sell the underlying asset. In an American option, the holder has the right to *exercise* at any time before or at maturity time T whereas an European option can be exercised only at the expiration date.

In practice most traded stock options are American. Moreover, many equity-linked insurance policies have American-like riders, i.e. insureds have additional *rights* that can be exercised at any convenient date for the policyholder.

A very important question that arises is *when an option buyer should exercise.* Unfortunately this is not an easy question to answer; we will come back to it later in the book when assuming a dynamic for the stock price. The focus of this section is rather to determine whether or not an American option should be exercised before maturity, and how early exercise affects an option price.

6.5.1 Lower bounds on American options prices

We will first compare European and American options that are otherwise identical, i.e. they are written on the same underlying asset, have the same strike price and maturity. To simplify the comparison, let us denote by P_t^A and P_t^E the time-t value of an American put and a European put, respectively. Also, let us denote by C_t^A and C_t^E the time-t value of an American call and a European call, respectively. We will use this notation temporarily, until the end of this section.[2]

Comparing European and American option prices

Intuitively, an American option should be worth at least as much as its European counterpart because an American option holder is able to exercise any time prior to or at maturity. As it gives more flexibility, it should have more value; otherwise there would be an arbitrage opportunity because a rational investor should not lose from having the possibility to exercise early. This is the case for both call and put options.

Indeed, assume instead that the European call is worth more than its American counterpart, i.e. assume $C_0^A < C_0^E$. In this case, we buy the American option, sell the European option and invest the (positive) difference $C_0^E - C_0^A$ at the risk-free rate. We hold on to the American call until maturity, i.e. we do not exercise early, so both options will offset each other at maturity.

2 In this book, and unless stated otherwise, the option is European.

We will thus end up with the profits generated by the risk-free investment. The same strategy works for put options and for any other time t between inception and maturity.

In conclusion, we have obtained the following relationships:

$$P_t^{\mathrm{A}} \geq P_t^{\mathrm{E}} \quad \text{and} \quad C_t^{\mathrm{A}} \geq C_t^{\mathrm{E}}, \tag{6.5.1}$$

for all $0 \leq t \leq T$.

Comparing with the exercise value

The **(early-)exercise value** or **intrinsic value** of an American option is the amount received upon exercise. Take, for example, an American put struck at K with expiration date T. The holder can decide at any time $0 \leq t \leq T$ to receive K in exchange for delivery of an asset worth S_t: the (early-)exercise value of this put is then $K - S_t$.

From equation (6.4.3) and equation (6.5.1), we have that

$$C_t^{\mathrm{A}} \geq C_t^{\mathrm{E}} \geq \max\{S_t - K\mathrm{e}^{-r(T-t)}, 0\} \geq S_t - K, \tag{6.5.2}$$

for any $0 \leq t \leq T$. In particular, we have $C_t^{\mathrm{A}} \geq S_t - K$. In other words, at any time t during its life, the price of the American call option is larger than its exercise value $S_t - K$.

This is also true for American put options: for any $0 \leq t \leq T$, we have $P_t^{\mathrm{A}} \geq K - S_t$. If, at some time t, we observe $P_t^{\mathrm{A}} < K - S_t$, then there is an arbitrage opportunity: we buy the American put and exercise it right away, making an immediate risk-less profit of $K - S_t - P_t^{\mathrm{A}} > 0$.

Note that we have not said that American option prices are *strictly* greater than their early-exercise values, as they could be equal. Those (random) times when the American option price equals its early-exercise value play an important role in the timing of early exercise. As the latter question will only be answered later, we now focus our attention on whether a rational investor should early-exercise an American call or put.

6.5.2 Early exercise of American calls

In equation (6.5.1), we have verified that an American call is always worth at least as much as its European counterpart, i.e. $C_t^{\mathrm{A}} \geq C_t^{\mathrm{E}}$. It would be natural to think/believe that this last inequality is in fact a strict inequality. The idea behind this belief is that the underlying stock S may reach its maximum value at some point in time during the life of the option, so that early exercise would be optimal at that time. In fact, this is never the case for American calls written on non-dividend-paying stocks:

> *It is never optimal to early-exercise an American call written on a non-dividend-paying stock.*

Indeed, even if an investor believes the stock price has reached its peak value at some time t, then the American call price C_t^{A} still is larger than its early-exercise value, as obtained above. Mathematically, for all t, we have

$$C_t^{\mathrm{A}} \geq S_t - K,$$

(see also equation (6.5.2)) no matter how large S_t is. This means that if the underlying stock S reaches a lifetime maximum, then the American call option price C^{A} will also contain this information and be worth more. Consequently, early-exercising an American call option at any time t is sub-optimal in the sense that we replace a financial position worth C_t^{A} by another position worth less, that is $S_t - K$.

For the compound annual ratchet, the computation is tedious, but we have

$$\max\left(I \times \prod_{k=1}^{10}(1+\beta \max(y_k, 0)), G_T\right) = (1+0.8\times 0.0222)\times(1+0.8\times 0.0204)\times \ldots$$
$$\times (1+0.8\times 0) = 1.332979.$$

Thus, the maturity benefit is

$$\max(100 \times 1.332979, 100) = 133.30.$$

Finally, the maximum value of the reference index is observed at time 9 (at the end of the 9-th year), i.e. $M_{10}^S = 1317.11$ and hence the high watermark benefit is

$$I \times \left(1+\beta R_T^{\max}\right) = 100(1+0.8\times 0.31771) = 125.3688. \qquad \blacksquare$$

8.3 Variable annuities

In what follows, we will use the name **variable annuity (VA)** for another popular sub-class of ELIA contracts. They are constructed with a *separate account*, also known as the *sub-account* of the contract, where the initial investment is deposited and credited with returns from a reference portfolio (underlying asset). The policyholder is then allowed to withdraw from this sub-account either for liquidity purposes during the accumulation phase or simply to purchase an annuity at retirement (annuitization phase). This sub-account is subject to a set of guarantees that apply at maturity or death, making VAs typical hybrids between insurance and investment.

One of the most important difference between EIAs and VAs is how guarantees are financed, i.e. how they are paid for by the policyholder. As mentioned before, in an EIA contract, the cost of the guarantee is paid implicitly through the participation rate β that lowers the upside potential. In a VA contract, premiums are withdrawn periodically from the sub-account and are usually set as a fixed percentage of the sub-account balance, acting again like a *penalty* on the credited returns. As before, these premiums also include management fees, operating expenses, taxes, etc. The sum of these expenses is known as the **management and expense ratio** (MER).[2]

The generic name for this type of ELIA contract is **separate account policy**. In the U.S., it is known as a VA, in Canada, as a *segregated fund*, and in the U.K., as a *unit-linked contract*. They borrow their name from typical regulations that prevent insurance companies from mixing assets backing variable annuities with other investments.

8.3.1 Sub-account dynamics

The insured's initial investment $A_0 = I$ is deposited in a sub-account. This sub-account is then credited with the returns of the reference portfolio. Also, it is from this sub-account that fees are deducted and that *policyholder withdrawals* will be made.

2 In the industry, the initial investment is often known as the *premium* whereas periodic contributions made by the policyholder to pay for the guarantee and other costs (MER) is known as a *fee* instead of a premium. We will keep using the terms *initial investment* and *periodic premium* so that the vocabulary is consistent with actuarial and financial mathematics.

Let the sub-account value at time k (after k months or years) be denoted by A_k, where $k = 1, 2, \ldots, n$. The sub-account value is adjusted periodically and it is not allowed to become negative so that deductions (withdrawals, fees) must be such that $A_k \geq 0$ for each k. With this notation, the index n corresponds to the maturity time T. More specifically, the sub-account dynamic is given by

$$A_k = A_{k-1} \times \frac{S_k}{S_{k-1}} \times (1 - \alpha) - \omega_k \quad \text{as long as } A_k \geq 0, \tag{8.3.1}$$

where α is the fee/premium rate deducted from the sub-account at the end of the period and where ω_k is the amount withdrawn by the policyholder, at the end of the k-th period (as long as it is less than the sub-account value prior to the withdrawal).

We can describe equation (8.3.1) as follows: the sub-account balance A_k is obtained by crediting returns $\frac{S_k}{S_{k-1}}$ to the previous sub-account balance A_{k-1}, then deducting the proportional periodic premium (done by multiplying by $(1 - \alpha)$) and finally deducting the policyholder's withdrawal ω_k. Again, unless stated otherwise, all the parameters and quantities are given and computed on an annual basis. Finding the *fair value* of α is a typical problem for insurance companies and is a concept related to finding the no-arbitrage price of an option: more details in Chapter 18.

If we set $A_{k-} = A_{k-1} \times \frac{S_k}{S_{k-1}} \times (1 - \alpha)$, i.e. if A_{k-} is the value of the sub-account just after deducing the fee but just before the k-th withdrawal, then we can rewrite (8.3.1) as follows:

$$A_k = A_{k-} - \omega_k \quad \text{if } \omega_k < A_{k-}.$$

Note that depending on the VA policy, there might be restrictions on withdrawals and guarantees might be adjusted based on the amounts withdrawn.

Example 8.3.1 *Sub-account balance after 1 year*

An investor puts \$100 in the sub-account tied to a VA policy. The periodic premium is set to 1% per annum. The investor plans on withdrawing \$5 at the end of the year, if the funds are available.

Assume the following scenario: the reference portfolio will grow by 4% over the next year. Let us compute the value of the sub-account balance at the end of the year, i.e. after the withdrawal, in this scenario.

Here, we have $A_0 = 100$, $\omega_1 = 5$, $\alpha = 0.01$. In this scenario, we have $S_1/S_0 = 1.04$ and therefore the sub-account balance at year-end would be

$$A_1 = A_0 \times \frac{S_1}{S_0} \times (1 - \alpha) - \omega_1 = 100 \times 1.04 \times 0.99 - 5 = 102.96 - 5 = 97.96.$$

Note that we were allowed to withdraw $\omega_1 = 5$ from the sub-account since $\omega_1 = 5 < A_{1-} = 102.96$.

In this scenario, in the second year, the return of the reference portfolio would be applied to a sub-account balance of 97.96. ■

Example 8.3.2 *Sub-account balance during a year*

A retiree puts \$120 in the sub-account tied to a VA policy. If the premium paid at the end of each month is 0.1% of the (monthly) balance and if the contract allows for withdrawals of

16.1 Model

As in the binomial and the trinomial models, the **Black-Scholes-Merton model** (BSM model) is assumed to be a *frictionless* market composed of two assets:

- a risk-free asset (a bank account or a bond) which evolves according to the risk-free interest rate r;
- a risky asset (a stock or an index) with a known initial value and unknown future values.

Trades in this market can occur at any *continuous* time $t \geq 0$ and therefore the BSM framework is a continuous-time market model. We assume that time is always expressed in years (even if it is possible to do otherwise) so that parameters are written on an annual basis. We will mostly work on a time horizon $[0, T]$ where T is very often the maturity of a derivative.

16.1.1 Risk-free asset

In the BSM model, there is an asset $B = \{B_t, 0 \leq t \leq T\}$ from which investors can earn an interest rate of r. This asset is said to be risk-free because capital and interest is repaid with certainty: there is no default. Thus, $r > 0$ is known as the risk-free rate and it is assumed to be continuously compounded, i.e. for $0 \leq t \leq T$,

$$B_t = B_0 e^{rt}.$$

As before, we set $B_0 = 1$. Note that, we could also have fixed $B_T = 1$ instead of B_0, in which case B would be modeling a zero-coupon bond.

16.1.2 Risky asset

In the BSM model, the price of the risky asset (often an index or a stock) $S = \{S_t, 0 \leq t \leq T\}$ evolves according to a geometric Brownian motion

$$S_t = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right\}, \tag{16.1.1}$$

where $W = \{W_t, 0 \leq t \leq T\}$ is a standard Brownian motion (with respect to the probability measure $\mathbb{P}$) and S_0 is a known quantity (initial asset price).

It follows from Section 14.5 that

$$S_t \overset{\mathbb{P}}{\sim} \mathcal{LN}\left(\ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right) t, \sigma^2 t\right)$$

i.e. for each time $0 < t \leq T$, the random variable S_t follows a lognormal distribution with parameters $\ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)t$ and $\sigma^2 t$ (with respect to the actuarial probability $\mathbb{P}$ measure).

Finally, note that the BSM model can be fully specified by four parameters: S_0, μ, r and $\sigma > 0$.

We can also express the model for the risky asset in terms of log-returns. On any given year, the log-return is such that

$$\ln(S_{t+1}/S_t) \overset{\mathbb{P}}{\sim} \mathcal{N}\left(\mu - \frac{\sigma^2}{2}, \sigma^2\right)$$

i.e. the continuously compounded annual return is normally distributed with mean $\mu - \frac{\sigma^2}{2}$ and variance σ^2. As a result, σ is called the volatility of the log-return.